

TOPIC TWO: FUNCTIONS

- Functions describe how one set of values is related to another. A function is a special type of relation in which every input has exactly one output.

Denoted as $f : A \rightarrow B$ // read as a function f from A to B

Meaning

Function f maps elements of set A to elements of set B .

For every $a \in A$, there must be exactly one corresponding value $f(a)$.

Note: A function takes one input, applies a rule and produces exactly one output.

Examples of functions

$f : \text{Customer} \rightarrow \text{CustomerID}$

$f : \text{Student} \rightarrow \text{StudentNo.}$

$f : \text{Employee} \rightarrow \text{Salary}$

Etc.

- **Function notation**

$f(x)$ is read as “ f of x ”

Example 1

Suppose $f(x) = 2x + 1$

Input: x	Rule: $2x + 1$	Output: $f(x)$
1	$2(1) + 1$	3
2		
3		
4		
5		

If the input is $x = 1$

$$f(1) = 2x + 1$$

$$= 2(1) + 1$$

$$= 3$$

Therefore $f(1) = 3$

Practice: Given that $x = 2$, find $f(x) = x^3$

- **Functions using sets**

Example 1

Let $A = \{1, 3, 5\}$. Define a function $f : A \rightarrow B$ where $f(x) = \frac{x}{3}$.

Solution

$$f(1) =$$

$$f(3) =$$

$$f(5) =$$

Example 2

Let $A = \{1, 2, 3\}$ and $B = \{2, 4, 6, 8\}$. Define a function $f : A \rightarrow B$ where $f(x) = 2x$.

Solution

$$f(1) =$$

$$f(2) =$$

$$f(3) =$$

$$\text{Therefore } f = \{(1, \quad), (2, \quad), (3, \quad)\}$$

Business Example 1

Suppose:

$E = \{E_1, E_2, E_3\}$ is a set of employees and $D = \{\text{IT, Finance, Marketing, HR, Security}\}$ is a set of all departments.

Define a function $f: E \rightarrow D$

A function can assign each employee to only one department.

Therefore; $E_1 \rightarrow \text{IT}$

$$E_2 \rightarrow \text{Finance}$$

$$E_3 \rightarrow \text{IT}$$

Example 2

An online shop can calculate the cost of every order, where

$f : \text{Order} \rightarrow \text{Cost}$

$$f(O_1) = 5000$$

$$f(O_2) = 10000$$

$$f(O_3) = 15000$$

- **Domain, Range and Codomain of a Function**

Consider $f : A \rightarrow B$

- a. The **domain** is the set of all possible inputs

Domain (f) = A

Let $A = \{101, 102, 103, 104\}$

Suppose these represent student registration numbers, then registration numbers form the domain.

- b. The **codomain** is the set containing the possible outputs.

Codomain (f) = B

If $B = \{\text{DBD}, \text{DCS}, \text{DAF}, \text{BBC}, \text{MBA}\}$

- c. The **range** is the set of output values that are actually produced by the function.

Example 1

$f = \{(101, \text{DBD}), (102, \text{DCS}), (103, \text{DAF}), (104, \text{BBC})\}$

Example 2: Given that $f = \{(101, \text{DBD}), (102, \text{DCS}), (103, \text{DAF})\}$. Is f a function? Explain your answer.

Qn: With a relevant example, explain how a function is different from a relation?

Class discussions: Importance of functions in business

Define a function which calculates the total sales of a product.

- **Representing Functions (read about)**

- **Types of Functions**

- 1. One-to-One Function**

A function is one-to-one (injective) if different inputs produce different outputs.

Example

$$f(x) = 2x$$

For $A = \{1, 2, 3\}$

$$f(1) = 2$$

$$f(2) = 4$$

$$f(3) = 6$$

Business Example

A unique employee ID can be mapped to one employee.

StudentNo $\rightarrow$ Student

Employee $\rightarrow$ Department

- 2. Many-to-One Function**

Several inputs may have the same output, but each input still has only one output.

Example

$$f(x) = x^2$$

For -2, -1, 0, 1, 2

x	-2	-1	0	1	2
f(x)	4	1	0	1	4

Business Example

Different customers can belong to the same customer category.

Customer $\rightarrow$ Category

C01 $\rightarrow$ High value

C02 $\rightarrow$ High value

C03 $\rightarrow$ Medium

- 3. Onto Function**

A function is onto (surjective) when every element in the codomain is actually used by at least one input.

Example

If $A = \text{patients}$

$\{P_1, P_2, P_3\}$ and

$B = \text{temperature levels}$

$\{\text{Low, Medium, High}\}$

Then;

Patient $\rightarrow$ Temperature level

$f(P_1) = \text{Low}$

$f(P_2) = \text{Medium}$

$f(P_3) = \text{High}$

4. Into Function

A function is into when at least one element of the codomain is not used.

Suppose $A = \text{customers}$ and $B = \text{ticket categories}$

Define a function

5. Constant Function

A constant function gives the same output for every input.

$$f(x) = c$$

Example: $f(x) = 1000$

$$f(1) = 1000$$

$$f(2) = 1000$$

$$f(3) = 1000$$

Business Example

Fixed school fees structure for all students

Fees (student) = 500000

In a supermarket, every customer receives the same incentive or reward.

6. Linear Functions

A linear function has the form: $f(x) = mx + c$, where

$m = \text{rate of change}$

$c = \text{starting value}$

Plot $f(x) = 2x + 1$ in Excel

Business Example

Suppose a delivery company charges

UGX 5,000 – fixed fee

UGX 1,000 - per kilometer

Then $C(d) = 1000d + 5000$ // d is the distance

Calculate delivery cost if $d = 10\text{km}$.

Practice: Model a revenue function

7. Quadratic Functions

A quadratic function has the form $f(x) = ax^2 + bx + c$

Example: Model of profit

$P(x) = 2x^2 + 100x - 500$, where x represents units sold

Investigate how profit changes as sales change

8. Piecewise Functions

A piecewise function uses different rules for different input ranges.

Example

A delivery company could charge

$C(d) = 2,000$	$0 < d \leq 2$
$2,000 + 1000(d - 2)$	$2 < d \leq 5$
$5,000 + 500(d - 10)$	$d > 10$

Meaning: $0 - 2 \text{ km} \rightarrow \text{UGX } 2,000$

Practice: Model a piecewise function for bulk pricing or mobile data pricing

- **Read about Identity and exponential functions**

- **Function composition**

Function composition means one function becomes the input of another.

Consider two functions $f(x)$ and $g(x)$

Function composition of f and g is written as: $(f \circ g)(x) = f(g(x))$

Read as “ f composed with g ”. Apply g first, then apply f

Example: Given that

$$f(x) = 2x + 3$$

$$g(x) = x^2$$

Find $(f \circ g)(x)$

Solution

i. Apply g

$$g(x) = x^2$$

ii. Put $g(x)$ into f

$$f(x) = 2x + 3$$

Replace x with $g(x)$

$$f(g(x)) = 2(x^2) + 3$$

$$\text{Therefore; } (f \circ g)(x) = 2x^2 + 3$$

Practice: Let $x = 2$

a. Find $(f \circ g)(x)$

b. Workout $(g \circ f)(x)$

Business Example

Suppose an event organizer sells tickets for a concert. The organizer calculates ticket revenue and then pays 10% in event taxes on the revenue. What is the overall amount received by the organizer?

Solution

i. Function g: Calculate the revenue from the tickets sales if x is the number of tickets sold and each ticket costs UGX30,000

$$\text{So } g(x) = 30,000x$$

ii. Function f: Find the ticket revenue after the 10% reduction

$$\text{So } f(x) = 0.9x$$

Therefore the overall amount received by the organizer is:

$$\begin{aligned} f(g(x)) &= 0.9(30,000x) \\ &= 27,000x \end{aligned}$$

Let $x = 500$

Find $f(g(x))$

Practice: Suppose MUBS was organizing a party for parents and guardians.

$f(x) = x + 10$, represents number of registered guests plus 10 VIP MUBS staff

$g(x) = 15,000x$ is the cost per meal

How much does MUBS spend on 100 guests?

- **Inverse Functions**

An inverse function reverses the operation of a function.

If $f(x) = x + 5$

Then $f^{-1}(x) = x - 5$

Let $x = 1$

$f(1) = 1 + 5$

$= 6$

Find inverse of $f(1)$

$f^{-1}(6) = 6 - 5$

$= 1$

Business Example

i. Suppose 1 USD = 3,927.74UGX

Define $f(x) = 3,927.74x$

Where x = amount in US Dollars

$f(x)$ = equivalent amount in UGX

If $x = 100\text{USD}$

Then $f(100) = 3,927.74(100)$

$= 392,774\text{UGX}$

ii. Convert 392,774UGX to USD.

$f(x) = 3,927.74x$

$f^{-1}(x) = \frac{x}{3,927.74}$

$$f^{-1}(392,774) = \frac{392,774}{3,927.74}$$

= ?

Class assignment: Collect the current USD-UGX exchange rate. Define a function that converts USD to UGX then derive the inverse function that converts UGX back to USD. Test your functions using three different amounts and verify that applying the function followed by its inverse returns the original amount.