

TOPIC ONE: RELATIONS

- **Relation**

A relation is a way of describing a connection between elements of two sets.

Example: If $A = \{a, b, c\}$ and $B = \{1, 2, 3\}$

Step 1: Find the Cartesian product

The Cartesian product $A \times B$ contains all possible ordered pairs or combinations.

$$A \times B = \{(a, 1), (a, 2), (a, 3), (b, 1), (b, 2), (b, 3), (c, 1), (c, 2), (c, 3)\}$$

Step 2: List only ordered pairs that satisfy a particular condition

A relation from A to B is any subset of $A \times B$

E.g. $R = \{(a, 1), (a, 2), (b, 1), (b, 2), (b, 3)\}$

Therefore: $R \subseteq A \times B$

This means that some elements of A are related to some elements in B.

- **Worked out Examples**

Example 1: Suppose $T = \{1, 4, 9, 16\}$ and $Y = \{1, 2, 3, 4\}$

What is the output of the relation “t is the square of y”?

Step 1

Cartesian product of $T \times Y = \{(1, 1), (1, 2), (1, 3), (1, 4), (4, 1), (4, 2), (4, 3), (4, 4), (9, 1), (9, 2), (9, 3), (9, 4), (16, 1), (16, 2), (16, 3), (16, 4)\}$

Step 2

$R = \{ \quad \quad \quad \}$

Example 2: From Example 1, find the relation “t is equal to y”.

Example 3: From Example 1, what is the relation “t is less than y”?

Example 4: From Example 1, list the elements of the relation “t is greater than y”.

Business Example

In real life, data is related e.g. customers are connected to products, students to courses, employees to departments, products to suppliers etc.

Consider $C = \{C01, C02, C03\}$ and $P = \{\text{lip gloss, perfume}\}$. Suppose C01 purchased lip gloss and perfume, C02 purchased perfume and C03 purchased perfume.

The relation **Purchased** can be written as:

$R = \{(C01, \text{lip gloss}), (C01, \text{perfume}), (C02, \text{perfume}), (C03, \text{perfume})\}$

This relationship can be used to determine:

- i. Popular products
- ii. Which customers buy similar products
- iii. Customer purchasing patterns
- iv. Products that are frequently bought together

Group discussion: Importance of relations in business intelligence and data analytics

- **Domain, Range and Codomain of a Relation**

Suppose the relation $R \subseteq A \times B$, where A is the first set and B is the second set.

- The domain is the set of all elements that appear as first components of the ordered pairs of the relation.
- The range is the set of all elements that appear as the second components of the ordered pairs of the relation.
- Codomain is the entire set B that the relation is defined to relate to.

Example

Let $A = \{1, 2, 3\}$ and $B = \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$. What is the domain, range and codomain of the relation “b is the square of a”?

- **Ways of Representing Relations**

- a. **Ordered pair**

A relation can be written as a set of ordered pairs. An ordered pair is a pair of elements written in a particular order using parentheses.

Example: Let $A = \{a, b\}$ and $B = \{x, y\}$

Ordered pairs are: $(a, x), (a, y), (b, x), (b, y)$

The relation $R = \{(a, x), (a, y), (b, x)\}$

Note: $(a, x) \neq (x, a)$

Qn: Is $(b, y) \in R$?

Business application: The order tells the relationship between the elements.

Example: Student $\longrightarrow$ **offers** $\longrightarrow$ Course

(S01, Computing Maths), (S01, Communication Skills), (S02, Computing Maths), (S02, Communication Skills)

b. Set-Builder notation

A relation can be described using a rule or condition.

Example: The relation $a < b$

$R = \{(a, b) \mid a, b \in A, a < b\}$

Meaning

R is the set of all ordered pairs (a, b) such that a and b belong to A where a is less than b .

Example: Suppose $A = \{1, 2, 3\}$,

Then $R = \{(a, b) \mid a, b \in A, a < b\}$ is

$R = \{ \quad \quad \}$

c. Arrow Diagram

An arrow diagram visually shows how elements are related.

Example:

$R = \{(1, 2), (1, 3), (2, 3)\}$

Draw the arrow diagram.

d. Graphs (Read about)

e. Table representation

Relations can be represented in a table

For $R = \{(1, 2), (1, 3), (2, 3)\}$

First element	Second element

f. Matrix

A relation can be represented using a matrix. If a relation exists indicate 1, 0 means the relation does not exist. Useful for social media analysis, recommendation systems etc.

Suppose students were asked to like Youtube videos on the E-learning Channel

Student	Video A	Video B
Elvis	1	1
Juliet	0	0
Adam	1	0
Vivian	1	0

Value 1 means student “likes” the video

g. Database Table Representation

Relations can be represented using database tables to show relationships between data. Useful for building reports and dashboards.

Example

Customers

CustomerID	CustomerName
C01	Daisy
C02	Comfort
C03	Felix
C04	Sulaina

Products

ProductID	ProductName
P01	Laptop
P02	Phone
P03	Tablet
P04	Headphones

The “Purchases” relation can be stored as:

Products

CustomerID	ProductID
C01	P02

C01	P04
C02	P04

Therefore: $R = \{ \quad \}$

Business insight???

- **Database Relationships**

1. **One-to-One Relation (1:1)**

Each element in one set is related to exactly one element in another set.

Example: Each Ugandan has exactly **one** NIN

2. **One-to-Many Relation (1:M or 1:N)**

One element in a set can be related to several elements in another set.

Examples

Useful for customer purchasing behavior analysis, order frequency analysis, recommendation systems, etc.

3. **Many-to-One Relation (M:1 or N:1)**

Many elements in one set can relate to one element in another set.

Examples

4. **Many-to-Many Relation (M:N)**

Many elements in a set can be related to many elements in another set.

Example: A customer can buy many products and also a product can be purchased by many customers

Therefore: Customer $\longleftrightarrow$ Product

Useful for analyzing customer preferences, product popularity, product combinations etc.

Food for thought: What is the relationship between you and your parent?

- **Class assignment**

Identify one relationship between different data in the business area.

Business Area	Relationship
Banking	Customer → <i>opens</i> → Account
HR	
Telecom	
Hospital	
E-commerce	
SACCO	
Supply chain	

Database Modelling Exercise

Scenario

SafeBoda needs to store information about customers, rides and payments. You will collect data from **5 students** in class and transform the information into a simple database.

Tasks

1. Collect data from 5 fellow students about their SafeBoda use.
2. Create the following database tables:

Customer Table

CustomerID	StudentName
C001	
C002	
C003	
C004	
C005	

Ride Table

RideID	CustomerID	PickupArea	Destination	Fare
R001				
R002				
R003				

Payment Table

PaymentID	RideID	PaymentMethod
P001		
P002		

P003		
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3. Determine the relationships:
 - i. One customer can have many rides
 - ii. Each ride has a corresponding payment record
4. Draw at least three insights from the results, such as:
 - i. most common payment method
 - ii. most frequently used pickup area
 - iii. average fare
 - iv. customer with the most rides
 - v. total amount spent by each customer

• Properties of Relations

A relation R on a set A is a subset of the Cartesian product i.e $R \subseteq A \times A$.

The properties of a relation describe how the elements of a set are connected to one another.

Let $A = \{1, 2, 3\}$ and suppose R is a relation on A .

1. Reflexive Relation

A relation R is reflexive if every element is related to itself i.e.

$(a, a) \in R$ for every $a \in A$

Example

$R = \{(1, 1), (2, 2), (3, 3), (1, 2)\}$

This relation is reflexive because the ordered pairs $(1, 1)$, $(2, 2)$, $(3, 3)$ are all in the relation.

Real life example

Consider the relation “has the same age as”

Every person has the same age as themselves.

Therefore the relation is reflexive

2. Irreflexive Relation

A relation R is irreflexive if no element is related to itself i.e.

$(a, a) \notin R$ for every $a \in A$

Example

$$R = \{(1, 2), (1, 3), (2, 3)\}$$

There are no pairs $(1, 1)$, $(2, 2)$, $(3, 3)$, therefore R is irreflexive.

Real life example

The relation “is older than”

No one is older than themselves. Therefore it is irreflexive.

3. Symmetric Relation

A relation R is symmetric if whenever a is related to b , then b is also related to a . i.e.

For every $a, b \in A$, if $a R b$ then $b R a$

Example

$$R = \{(1, 2), (2, 1), (2, 3), (3, 2)\}$$

Since $(1, 2) \in R$ then $(2, 1)$ must also $\in R$, similarly $(2, 3)$, $(3, 2)$ are in the relation. Therefore it is symmetric.

Real life example

The relation “is a sibling of” is symmetric.

If Simon is a sibling of Comfort, then...

4. Read about Asymmetric Relation

5. Antisymmetric Relation

A relation R is antisymmetric if two different elements cannot relate to each other in both directions unless the element is the same i.e. $a = b$

This means that if $a \neq b$, then it is impossible to have both (a, b) and (b, a) in the same relation.

Example

The relation “is less than or equal to” is antisymmetric if $a \leq b$ and $b \leq a$, then $a = b$

$R = \{(1, 1), (1, 2), (2, 2), (2, 3), (3, 3)\}$ is antisymmetric because $(2, 1)$ and $(3, 2)$ are not in the relation.

Real life example

The relation “is supervisor of” is antisymmetric

If Derrick is a supervisor of Joseph, can Joseph be a supervisor of Derrick

6. Transitive Relation

A relation R is transitive if a relates to b , and b relates to c , then a must relate to c .

If $(a, b) \in R$ and $(b, c) \in R$, then $(a, c) \in R$.

The relation “is less than” is transitive. For any real numbers a, b, c , if $a < b$ and $b < c$, then $a < c$.

Consider a set of numbers and define a relation “is divisible by” as follows

12 is divisible by 3

3 is divisible by 1

Write the relation and determine if it is transitive.

Real life examples

“is taller than”

“a sub-category of”

“reports to”

“is ancestor of”

“is located north of”

• Equivalence Relation

A relation R on a set is an equivalence relation if it satisfies three properties i.e. reflexivity, symmetry and transitivity. Elements are grouped into equivalence cases based on their similarity or equality.

Example

Consider set $A = \{1, 2, 3\}$ and a relation $R = \{(1, 1), (1, 2), (1, 3), (2, 1), (2, 2), (2, 3), (3, 1), (3, 2), (3, 3)\}$. Is R an equivalence relation?

Solution

Show that R is reflexive, symmetric and transitive.

- Reflexive: Check for existence of every aRa pair in the relation for all elements present in the set.
- Symmetric: Check for existence of every bRa pair for every aRb in the relation.

- Transitive: For every aRb and bRc pairs in the relation, check if aRc is also present in R

If all three properties are satisfied, then the relation is an equivalence relation.

Real life example

Suppose we have four users $A = \{\text{Stacy, Tracy, Cassy}\}$

Define R as “two users follow exactly the same accounts”

Suppose:

User	Account followed
Stacy	@News, @Travel, @Music
Randy	@News, @Travel, @Music
Tracy	@Travel, @Music

Determine whether R is an equivalence relation.

- **Reflexive**

Every user follows the same accounts as themselves

i.e. $\text{Stacy } R \text{ Stacy}$

Therefore R is reflexive

- **Symmetric**

If Stacy follows exactly the same accounts as Randy, then Randy follows exactly the same accounts as Stacy

Therefore R is symmetric

- **Transitive**

If Stacy follows exactly the same accounts as Randy and Randy follows exactly the same accounts as Tracy, then Stacy follows exactly the same accounts as Tracy

Therefore, the relation “two users follow exactly the same accounts” is an equivalence relation because it is reflexive, symmetric and transitive.

- **Practice Example**

Show that the relation R in the set $A = \{1, 2, 3, 4, 5\}$ given by $R = \{(a, b) : |a - b| \text{ is even}\}$ is an equivalence relation.

- **Partial Order Relation on a Set (POSET)**

A relation R on a set A is called a partial order relation if it is reflexive, anti-symmetric and transitive.

Example 1

Consider set $A = \{a, b\}$

$R = \{(a, a), (b, b), (a, b)\}$ is a partial order relation because $(b, a) \notin R$.

$R = \{(a, a), (b, b), (a, b), (b, a)\}$ is...because...

Example 2

Consider set $R = \{(1, 1), (1, 3), (1, 4), (2, 1), (2, 3), (2, 4), (3, 3), (3, 4), (4, 4), (4, 3)\}$. Verify that R is a partial order relation.

Step 1: Check if the relation is reflexive

Step 2: Check if the relation is anti-symmetric

Step 3: Check if the relation is transitive

Find a relevant business example where POSET can be applied.

- **Hass Diagram**

Read about a Hass diagram