

Limits at Infinity

When asked the limits at infinity, we first divide through the function by the highest power of x ; then find the limit of the function.

Recall: $\lim_{x \rightarrow \infty} \frac{a}{x^n} = 0$ where a and n are constants.

For instance,

$$\lim_{x \rightarrow \infty} \frac{1}{x}; \frac{1}{10}, \frac{1}{1000}, \frac{1}{10000}, \frac{1}{1000000} \approx 0$$

Also,

$$\lim_{x \rightarrow \infty} \frac{1}{x^2}; \frac{1}{100}, \frac{1}{1000000}, \frac{1}{100000000}, \text{ etc } \approx 0$$

Therefore, we say $\lim_{x \rightarrow \infty} f(x) = L_1$ (finite number) if the value of x becomes very big.

Also $\lim_{x \rightarrow -\infty} f(x) = L_2$ (finite number) if the value of x becomes very small.

Examples: Find:

$$1. \lim_{x \rightarrow \infty} \frac{x+1}{x}$$

Divide all through by the highest power of x

$$\begin{aligned} \lim_{x \rightarrow \infty} \frac{x+1}{x} &= \lim_{x \rightarrow \infty} \left(\frac{x}{x} + \frac{1}{x} \right) \\ &= \lim_{x \rightarrow \infty} \left(1 + \frac{1}{x} \right) = 1 + 0 = 1 \end{aligned}$$

$$2. \lim_{x \rightarrow \infty} \frac{x^2 + 3x + 1}{3x^2 + x + 2}$$

Divide all through by x^2 (the highest power of x)

$$\lim_{x \rightarrow \infty} \frac{x^2 + 3x + 1}{3x^2 + x + 2} = \lim_{x \rightarrow \infty} \frac{\frac{x^2}{x^2} + \frac{3x}{x^2} + \frac{1}{x^2}}{\frac{3x^2}{x^2} + \frac{x}{x^2} + \frac{2}{x^2}}$$

$$= \lim_{x \rightarrow \infty} \frac{1 + \frac{3}{x} + \frac{1}{x^2}}{3 + \frac{1}{x} + \frac{2}{x^2}}$$

$$= \frac{1 + 0 + 0}{3 + 0 + 0} = \frac{1}{3}$$

Exercise: Compute the:

1. $\lim_{x \rightarrow \infty} \frac{x^2 + 4x + 1}{-4x^2 - 6}$

2. $\lim_{x \rightarrow -\infty} \frac{4x^2 - 2x}{7x^3 + 8x^2}$