

TOPIC ONE: SETS

- **Set**

A set is an unordered collection of distinct objects, called members or elements of a set.

Examples

$A = \{2, 4, 6, 8, 10\}$

$C =$ all employees of the university

$L =$ products with more than 100 reviews

- **Elements of a set**

$a \in A$ means that a is an element in the set A or a is a member of A

$b \notin A$ means b is not an element in the set A b is not a member of A

- **Set Representation**

1. **Roster notation:** Elements are listed in braces. E.g.

Products = {Laptop, phone, Tablet, Printer}, $A = \{1, 2, 3, \dots, 9\}$

2. **Descriptive form:** Set is described in words. E.g.

A is a set of all products available in the online shop

3. **Set builder notation:** Elements are characterized by stating the properties they must have to be members.

General form is $\{x \mid x \text{ has property } P\}$ and is read “the set of all x **such that** x has property P .”

$S = \{x \mid x \text{ is a positive counting number less than } 10\}$

$K = \{x \mid x = \text{customer whose monthly purchases exceed UGX } 1,000,000\}$

$T = \{t \mid t \text{ is a square number, } 4 \leq t < 36\}$ // read comma as where

- **Cardinality of a set**

Cardinality means number of elements in a set.

Example: $A = \{a, b, c, d\}$

$$| A | = 4$$

- **Types of Sets**

- i. **Empty set**

A set containing no elements. Symbol is $\emptyset$.

Example: A set of customers who purchased a product that the business does not sell.

- ii. **Singleton set**

A set containing one element. E.g. set of an even prime number

- iii. **Finite set**

A set containing a fixed or countable number of elements. E.g. all products in stock, customers who purchased a car spare part, etc.

- iv. **Infinite set**

A set with infinitely many elements. E.g. $N = \{1, 2, 3, \dots\}$, all possible future customers etc.

- v. **Equal sets**

Two sets are equal if they contain exactly the same elements.

Example: Given $A = \{1, 2, 3\}$ and $B = \{2, 1, 3\}$

Therefore: $A = B$

- vi. **Equivalent sets**

Two sets are equivalent if they have the same number of elements even if the elements are different. Therefore equal sets can be equivalent sets.

Examples...

- vii. **Disjoint sets**

Two sets are disjoint when they have no elements in common.

Example: $A = \{1, 2, 3\}$ and $B = \{5, 6, 7\}$

Therefore: $A \cap B = \emptyset$

A set of students offering DBIDA and a set of students offering DBC are disjoint sets.

Used to create mutually exclusive categories in business.

- **Universal set**

A universal set contains all elements in a set. E.g. U = all lecturers in the computing faculty

- **Subsets of a set**

A set A is a subset of B if every element of A is also an element of B .

Denotes as: $A \subseteq B$

Example

Let $A = \{1, 2, 3\}$ and $B = \{2, 1, 3\}$

$A = B$

Therefore: $A \subseteq B$ and $B \subseteq A$

- **Proper subset**

A proper subset is a subset that contains some, but not all of the elements of another set.

Example: $Y = \{\text{apple, banana, watermelon}\}$ and $X = \{\text{apple, banana}\}$

Set X is a subset of set Y but $X \neq Y$.

Therefore: $X \subset Y$

Example in business

- **Listing the subsets of a set**

Let $B = \{1, 2\}$

Subsets of B are: $\emptyset, \{1\}, \{2\}, \{1, 2\}$

Note: Every non empty set has at least two subsets i.e. $\emptyset$ and itself.

- **Number of subsets of a set**

If a set has n elements, it has 2^n subsets where n is the number of elements in the set.

Example...

- **Power set**

A power set of a set is the set of all possible subsets of that set.

Power set of A is written as $P(A)$. If a set has n elements, its power set has 2^n subsets.

Example: Suppose $A = \{a, b\}$

Subsets of A are: $\emptyset, \{a\}, \{b\}, \{a, b\}$

Therefore: $P(A) = \dots$

Recommendation systems can use these combinations to study which products users tend to select together.

- **Set Operations**

- i. **Union**

A union of A and B contains all elements belonging to A, B or both i.e. at least one. Useful for combining data, measuring totals, avoiding duplication

Example in business

- ii. **Intersection**

The intersection contains elements common to both sets. Useful for cross-selling campaigns.

Example in business

- iii. **Difference of sets**

The difference $A - B$ contains elements in A that are not in B.

Example: $A = \{1, 2, 3, 4\}$ and $B = \{3, 1, 4, 5\}$

$A - B =$

Example in business

Useful for...

- iv. **Complement**

The complement of A consists of elements in the universal set that are not in A.

Business application: Given that U = all registered MM customers and A = all customers who have bought monthly data bundle this month

What is A^c ?

Useful for...

- **Venn diagrams**

A Venn diagram is a visual representation of sets using circles inside a rectangle representing the universal set.

Suppose:

U = all DBIDA students

A = all students who own a laptop

B = all students who own a mobile phone

The rectangle represents U, while the circles represent A and B.

Try listing elements in each set

Parts on a Venn diagram

If $A = \{1, 2, 3, 4, 5\}$ and $B = \{3, 4, 5, 6, 7\}$

i. A only

Elements in A AND NOT in B.

$A - B$ or $A \cap B^c$

ii. B only

Elements in B AND NOT in A.

$B - A$ or $A^c \cap B$

iii. Intersection

Elements in both sets.

$A \cap B$

iv. Union

Elements which belong to A, B OR both.

$A \cup B$

v. Neither

Elements neither in A nor B.

$(A \cup B)^c$

Draw a Venn diagram showing A only, B only, $A \cap B$, $A \cup B$ and $(A \cup B)^c$.

• Calculations from a Venn diagram

Example 1: Suppose a company has:

- 1,000 customers
- 600 bought Product A

- 400 bought Product B
- 200 bought both

So:

$$|U| =$$

$$|A| =$$

$$|B| =$$

$$|A \cap B| =$$

- Draw the Venn diagram
- Find the number who bought A or B.

$$\begin{aligned} |A \cup B| &= |A| + |B| - |A \cap B| \\ &= 600 + 400 - 200 \\ &= 800 \end{aligned}$$

Therefore, 800 customers bought at least one of the two products.

Note: Intersection is subtracted because the customers were already counted once.

- How many customers bought neither?
- Find $|A - B|$
- What is B' ?
- What is the output of $|A \cap B'| + |A \cap B| + |B \cap A'|$?

Example 2: Suppose

V = customers use MTN Mobile money

X = customers use Airtel Money

Y = customers use a bank mobile app

Draw a Venn diagram using the following information.

$ V $	60
$ X $	50
$ Y $	40
$ V \cap X $	20
$ V \cap Y $	15
$ X \cap Y $	10
$ V \cap X \cap Y $	5
$ V \cup X \cup Y $	?
$ U $	?

In summary;

$$|A - B| = |A| - |A \cap B|$$

$$|A^c| = |U| - |A|$$

$$|A \cup B| = |A| + |B| - |A \cap B|$$

$$|A \text{ only}| = |A| - |A \cap B|$$

$$|B \text{ only}| = |B| - |A \cap B|$$

$$|(A \cup B)^c| = |U| - |A \cup B|$$

$$|A \cup B \cup C| = |A| + |B| + |C| - |A \cap B| - |A \cap C| - |B \cap C| + |A \cap B \cap C|$$

- **Cartesian product**

The Cartesian product of A and B is $A \times B$. It consists of all possible ordered pairs where $a \in A$ and $b \in B$.

Example 1:

If $A = \{\text{student1}, \text{student2}\}$ and $B = \{\text{CM}, \text{BCS}\}$, then:

$$A \times B = \{(\text{student1}, \text{CM}), (\text{student1}, \text{BCS}), (\text{student2}, \text{CM}), (\text{student2}, \text{BCS})\}$$

This represents total enrollments in a university

Example 2:

- Find the Cartesian product of $A \times A$ given that $A = \{1, 2, 3\}$.
- How many ordered pairs are in the Cartesian product?

Note: The cardinality of the Cartesian product is given by;

$$|A \times B| = |A| \times |B|$$

- **Group Discussion**

With relevant examples from different industries, discuss the importance of sets.

MCQs

1. Let C = employees in the computing department

If Jerry is an employee in the computing department, which of the following is correct?

- A. $\text{Jerry} \in C$ B. $C \in \text{Jerry}$ C. $\text{Jerry} = C$ D. $\text{Jerry} \notin C$

2. Which of the following is most likely an infinite set?

- A. All students enrolled in a DBIDA class
B. All possible whole-number transaction IDs
C. All teaching staff at MUBS

- D. All products in Capital Shoppers supermarket
3. Which notation correctly represents a set of cities?
- $Y = [\text{Nairobi, Kampala, London, New York}]$
 - $Y \{ \text{Nairobi, Kampala, London, New York} \}$
 - $Y = (\text{Nairobi, Kampala, London, New York})$
 - $Y = \{ \text{Nairobi, Kampala, London, New York} \}$
4. In customer analytics, the union of customers who use mobile money services and those who use mobile banking services can help a business determine:
- Customers who use neither payment method
 - The total customers using at least one of the two payment methods
 - The total number of customers
 - Customers using both methods only
5. In a school database, the complement of the set of students in S.6 represents:
- Students in S.6 only
 - Students not in S.6
 - All students in the school
 - Students not in S.6 but are in the school
6. An online shopping recommendation system tracks three products:
 $S = \{\text{laptop, mouse, headphones}\}$
 The system uses subsets of S to analyze different combinations of products that customers may purchase. How many possible product combinations are represented in $P(S)$?
- A. 3 B. 6 C. 8 D. 9
7. Which business intelligence task best demonstrates the use of sets?
- Designing a company logo
 - Identifying customers who belong to multiple customer groups
 - Typing a business letter
 - Changing the color of a dashboard
8. A data analyst wants to identify students who applied for DBIDA and not DBA. Which operation should be used?
- A. $X \cup Y$ B. $X \cap Y$ C. $X - Y$ D. X^I

9. If a business has 80 customers who bought Product A, 60 customers who bought Product B and $|A \cap B| = 20$. How many customers bought Product A or B?
 A. 100 B. 120 C. 140 D. 160
10. Given that $T = \{2, 4, 6, 8\}$ and $Q = \{3, 9, 12, 15\}$. Which statement best describes sets T and Q?
 A. $T \cap Q = \emptyset$
 B. $T \neq Q$
 C. $T \equiv Q$
 D. All the above
11. What is the correct set builder form of $\{5, 10, 15, 20\}$?
 A. $\{x \mid x = 5n\}$
 B. $\{x \mid x \text{ is a multiple of } 5, 5 \leq x < 25\}$
 C. $\{x \mid x \text{ is a multiple of } 5, x < 25\}$
 D. $\{x \mid x \text{ is a multiple of } 5, 5 \leq x \leq 25\}$
12. If: $|A| = 3$ and $|B| = 4$, what is $|A \times B|$?
 A. 7 B. 12 C. 1 D. 24

• Laws of Sets

Business analytics deal with groups of data that can be represented as sets.
 Examples

A = students who passed an exam

B = transactions above a certain value

C = customers who frequently use Mobile Money etc.

Laws of sets or set properties help in filtering data, data cleaning, recommendation systems, customer segmentation etc.

1. Identity Laws

Combining a set with the empty set or universal set leaves the set unchanged.

Suppose A = active Wi-Fi users

i. Union identity

$$A \cup \emptyset = A$$

Adding an empty customer group does not change the original group

- ii. Intersection identity

$$A \cap U = A$$

Every element of A already belongs to the universal set, so selecting customers in A from a large group still gives A.

2. Domination Laws

These laws show what happens when a set is combined with the universal set or empty set.

- i. Union

$$A \cup U = U$$

Example: If any employee group is combined with all employees in a university, the result is all employees.

- ii. Intersection

$$A \cap \emptyset = \emptyset$$

If no elements exist in one group, there can be no common elements,

3. Commutative Laws

The commutative property means that changing the order of the set does not change the result.

- i. $A \cup B = B \cup A$

- ii. $A \cap B = B \cap A$

Example: If A = students who passed CM and B = students who passed ITE, What do the following mean?

$$A \cup B, \quad B \cup A, \quad A \cap B, \quad B \cap A$$

Therefore $A \cup B = B \cup A$

So the order in which two groups are combined does not affect the final group.

4. Associative Laws

The associative property is used to combine more than two sets and the grouping does not matter.

- i. $(A \cup B) \cup C = A \cup (B \cup C)$

- ii. $(A \cap B) \cap C = A \cap (B \cap C)$

Example: Let

A = Facebook users

B = Instagram users

C = Tiktok users

Customers can be grouped in different ways without changing the final result.

5. Distributive Laws

These laws show how union and intersection can be distributed across each other.

i. First distributive law

$$A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$$

This means a customer belongs to A and belongs to either B or C.

This is equivalent to;

Customers in both A and B, or customers in both A and C.

Example

Let A = customers who purchased online

B = customers who paid using MM

C = customers who paid using internet banking

What does $A \cap (B \cup C)$ mean?

ii. Second distributive law

$$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$$

Customers who purchased product A or customers who purchased both product B and C.

Prove the equivalence if $A = \{1, 2, 3, 4\}$, $B = \{3, 4, 5, 6\}$ and $C = \{4, 5, 6, 7\}$

6. Absorption Laws

i. First absorption law

$$A \cup (A \cap B) = A$$

Example

A = all customers who bought sugar

B = customers who used MM

So $A \cap B$ represents customers who bought..., but all those customers are already inside A.

ii. Second absorption law

$$A \cap (A \cup B) = A$$

Every element in A is already contained in $A \cup B$. Therefore if A is selected from a larger group that already contains A, the result is A.

7. De Morgan's Laws

i. First De Morgan's Law

$$(A \cup B)^I = A^I \cap B^I$$

The customers who are not in A or B are the same as customers who are not in A and not in B.

ii. Second De Morgan's Law

$$(A \cap B)^I = A^I \cup B^I$$

What is the meaning?

Read about

Idempotent laws, complement laws and double complement laws