BBM 3209
OPERATIONS RESEARCH

DECISION ANALYSIS

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Decision Theory

Background

- All managers are decision-makers.
- Managers must understand how decisions are made and know what decision-making tools are available.
- Usually, the success or failure that people and companies experience depends on the quality of their decisions.

Not only managers, but everybody makes decisions

Background – Decision Process

1) Define the problem and the factors that influence it

   - Must be clear and concise.

2) Establish decision criteria and goals

   - Develop specific and measurable objectives

3) Formulate a model or relationship between goals and variables.

   - Models contain one or more variables (Variables are measurable quantities that are subject to change).

Background – Decision Process

4) Identify and evaluate alternatives.

   - Generate as many solutions to the problem as possible (and usually quickly).

5) Select the best alternative

   - Usually best satisfies and is most consistent with the stated goals.

6) Implement the decision

   - This stage is sometimes the most challenging (Involves e.g. task assignments and a timetable for implementation)

What is a decision?

- A decision is the **conclusion** of a **process** designed to weigh the relative utilities or merits of a set of **available alternatives** so that the most **preferred course** of action can be selected for implementation.
  - *identify the most preferred choice to satisfy the desired goal or objective*

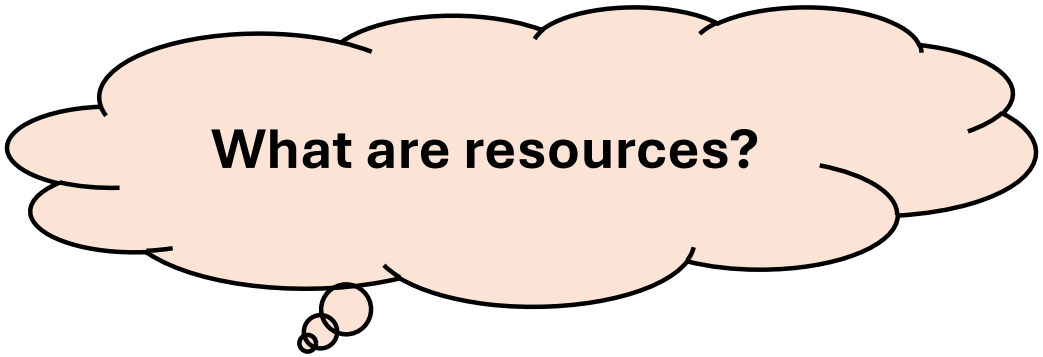
Why must decisions be made?

Resources are limited

Wants and needs are

- **unlimited**
- **diversified**

Why must decisions be made?



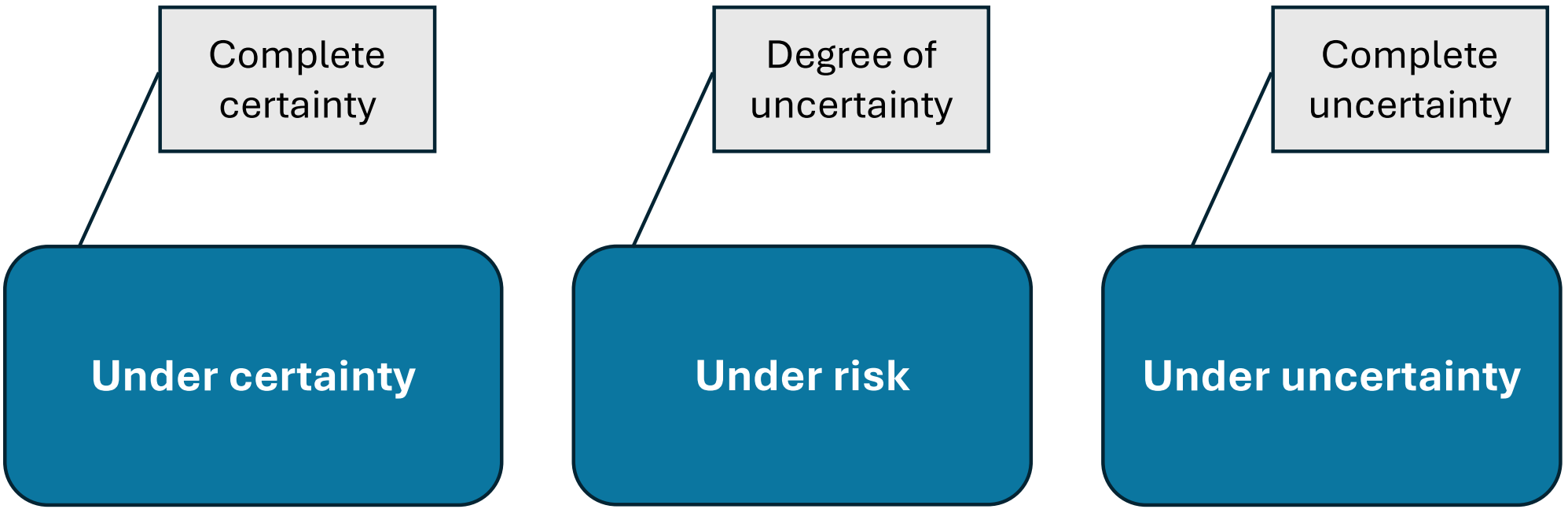
Resources are limited

Wants and needs are

- **unlimited**
- **diversified**

Decision-making environments

What is known?



Steps in the decision theory approach

- List the viable alternatives (strategies) that can be considered in the decision.

- List all future events that can occur.
  - *These future events (not in the control of the decision maker) are called states of nature.*

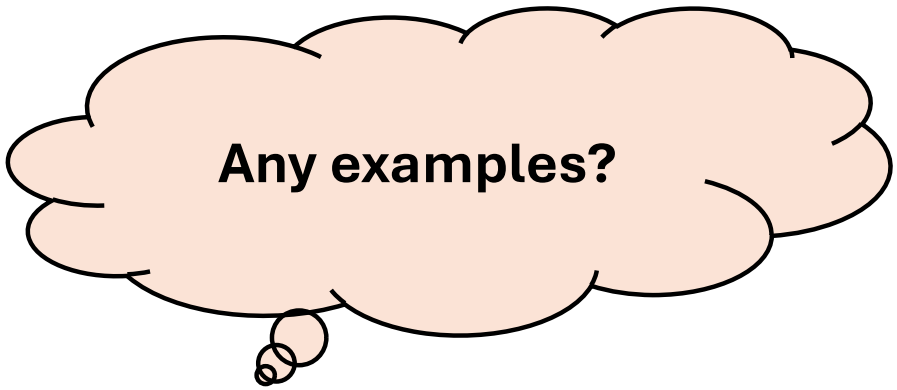
- Construct a payoff table for each possible combination of alternative courses of action and state of nature.

- Choose the criterion that results in the largest payoff (??) - monetary value or satisfaction
  - *Subject to the decision-making environment*

Decision making under certainty

- Decision making under certainty **assumes** that all relevant information required to make a decision is **certain** in nature and is well known.
  - *Complete knowledge, stability, and no ambiguity*
- Available strategies and their payoffs, and each strategy will have a unique payoff, resulting in certainty.
- The decision-making may be of a single objective or of multiple objectives

Class discussion



Class exercise 1

ABC Corporation wants to launch one of its mega campaigns to promote a special product. The promotion budgets have not yet been finalized, but they know that some Ugs. 5 000 000 is available for advertising and promotion.

Management wants to determine the optimal budget for television spots, considering which medium is most suitable for their product. They have created five 'T.V. campaign strategies' with their projected outcome in terms of an increase in sales. The required data is provided in the table below.

Determine which strategy should be selected to maximize utility.

(max $\frac{B}{C}$)

Cost (C)

Benefits (B)

Strategy	Cost in 1000 Ugs	Increase in sales in 1000 Ugs
A	1 800	1 780
B	2 000	2 020
C	2 250	2 420
D	2 750	2 680
E	3 200	3 240

Class exercise 1

Strategy A $\Rightarrow \frac{1780000}{1800000} = 0.989$

$\left(\frac{B}{C}\right)_{max}$

$B = \frac{2020000}{2000000} = 1.010$

$C = \frac{2420000}{2250000} = 1.076$ ✓ Maximum utility

$D = \frac{2680000}{2750000} = 0.975$

$E = \frac{3240000}{3200000} = 1.012$

The criterion for selecting the strategy (for maximum utility) is to choose the strategy that yields the maximum utility, i.e., the highest ratio of outcome (increase in sales) to cost.

Class exercise 2

Consider an XYZ company, which is developing its annual plans in terms of three objectives:

i. Increased profits
ii. Increased market share
iii. increased sales.

The company has formulated three strategies to achieve the stated objectives. The table below gives the relative weightage of objectives and scores the project.

Find the optimal strategy that yields maximum weighted or composite utility

Three objectives	**Profit**	**% increase (Market share)**	**% increase (Sales growth)**
	N_1	N_2	N_3
Weights	**0.2**	**0.5**	**0.3**
Strategy			
S_1	7	4	9
S_2	3	6	7
S_3	5	5	10

Class exercise 2

$S_1 = (0,2 \times 7) + (0,5 \times 4) + (0,3 \times 9) = 6.1$

$S_2 = (0,2 \times 3) + (0,5 \times 6) + (0,3 \times 7) = 5.7$

$S_3 = (0,2 \times 5) + (0,5 \times 5) + (0,3 \times 10) = 6.6$ ✓

Maximum utility $\Rightarrow 6.6 \rightarrow S_3$

Decision making under risk

- Each strategy results in more than one outcome or payoff, and a probability measure is attached to the payoffs.

- The criterion for evaluating decision alternatives (courses of action) under risk is the *Expected Monetary Value* (EMV) or *Expected Utility*.

- The EMV is determined from

$$EMV\left(course\ of\ action, S_j\right) = \sum_{i=1}^{m} p_{ij} p_i$$

Where,

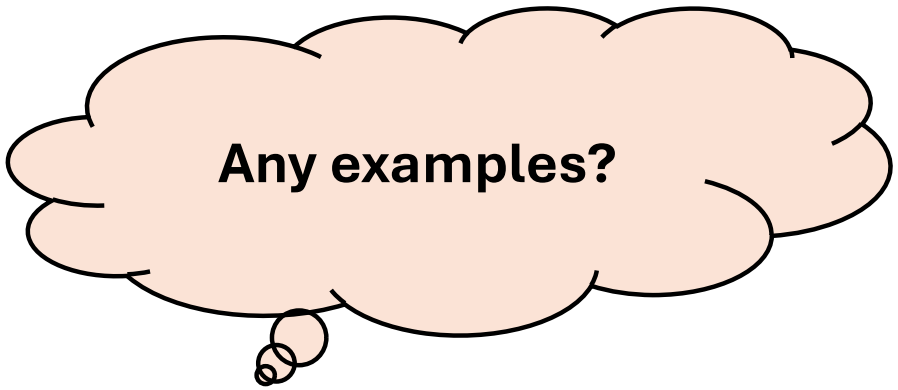
m = number of possible states of nature

p_i = Probability of occurrence of state of nature, N_i

p_{ij} = Payoff associated with state of nature N_i and course of action/strategy, S_j

Decision making under risk

- The optimal strategy is the strategy with the highest expected utility (or highest expected monetary value (EMV)).



Class exercise 3

A marketing manager of an insurance company has kept complete records of the sales efforts of the sales personnel. These records contain data regarding the number of insurance policies sold and net revenues received by the company as a function of four different sales strategies.

The manager has constructed the conditional payoff matrix given below, based on his records. (The state of nature refers to the number of policies sold). The number within the table represents utilities. Suppose you are a new salesperson and that you have access to the original records as well as the payoff matrix.

Which strategy would you follow?

State of nature	N_1	N_2	N_3
Probability	**0.2**	**0.5**	**0.3**
Strategy ↓	Utility	Utility	Utility
S_1(1 call, 0 follow-up)	4	6	10
S_2 (1 call, 1 follow-up)	6	5	9
S_3 (1 call, 2 follow-ups)	2	10	8
S_4 (1 call, 3 follow-ups)	10	3	7

Class exercise 3

$S_1 = (0,2 \times 4) + (0,5 \times 6) + (0,3 \times 10) = 6.8$

$S_2 = (0,2 \times 6) + (0,5 \times 5) + (0,3 \times 9) = 6.4$

$S_3 = (0,2 \times 2) + (0,5 \times 10) + (0,3 \times 8) = 7.8$ ✓

$S_4 = (0,2 \times 10) + (0,5 \times 3) + (0,3 \times 7) = 5.6$

$S_3 \Rightarrow 7.8$

Class exercise 4

A company is planning for its sales targets and the strategies to achieve these targets. The data in terms of three sales targets, their respective utilities, various strategies, and appropriate probability distribution are given in the table given below.

What is the optimal strategy?

Sales targets (1000)	50	75	100
Utility	**4**	**7**	**9**
	Probability	Probability	Probability
Strategy ↓			
S_1	0.6	0.3	0.1
S_2	0.2	0.5	0.3
S_3	0.5	0.3	0.2

Class exercise 4

$S_1 = (50 \times 0,6) + (75 \times 0,3) + (100 \times 0,1)$

$= 62.5$

$S_2 = (50 \times 0,2) + (75 \times 0.5) + (100 \times 0,3)$

$= 77.5$

$S_3 = (50 \times 0,5) + (0,3 \times 75) + (100 \times 0,2)$

$= 67.5$

$S_2 \Rightarrow 77.5$ ✓

$S_1 = (0,6 \times 4) + (0,3 \times 7) + (0,1 \times 9)$

$= 5.9$

$S_2 = (0,2 \times 4) + (0,5 \times 7) + (0,3 \times 9)$

$= 7.0$ ✓

$S_3 = (0,5 \times 4) + (0,3 \times 7) + (0,2 \times 9)$

$= 5.9$

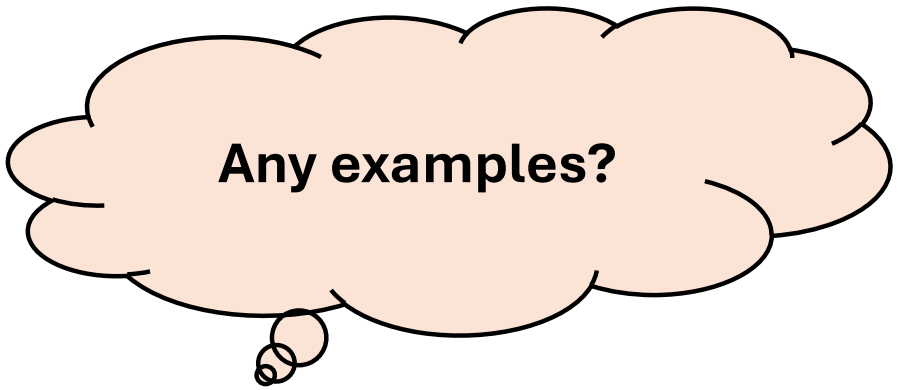
$$\textbf{Expected monetary value } of\ a\ strategy = \sum Sales\ target\ * probabiity$$

$$\textbf{Expected utility } of\ a\ strategy = \sum Utility * probabiity$$

$S_2 = 7.0$ ✓

Decision making under uncertainty

- Decision making under uncertainty is formulated exactly in the same way as decision making under risk, but
  - *No probability to each strategy is attached*.



Decision making under uncertainty

- Criteria of decision-making under uncertainty:
  - Optimism criterion
  - Pessimism criterion
  - Regret (salvage) criterion
  - Equal probabilities (Laplace) criterion
  - Coefficient of optimism (Hurwiez) criterion

Optimism and pessimism criteria

❑ Optimism criterion

- Determine the best possible outcome in each strategy
- Identify the best of the best outcome to select the optimal strategy.

❑ Pessimism criterion

- Determine worst possible outcome in each strategy (row minimums),
- Select the best of the worst outcomes to select the optimal strategy.

Illustrations

	W_1	W_2	W_3	max	min
S_1	15	12	18	18	12
S_2	9	14	10	14	9
S_3	13	4	26	26	4

Optimum $\Rightarrow$ Maximax $\Rightarrow 26 \Rightarrow S_3$

Pessimism $\Rightarrow$ maximin $\Rightarrow 12 \Rightarrow S_1$

Regret criterion

- Determine the regret matrix or opportunity loss matrix
  - identify the highest element and then subtract all the individual elements of that column, cell by cell, from the highest element to obtain the corresponding column of the regret matrix
- First, determine the maximum regret
- Identify the maximum of the maximum regret values
- Select the minimum of the maximum regret (Minimax regret).
- *If 2 strategies have the same minimax, then additional factors may be needed for selection*

Illustrations

	N_1	N_2	N_3
S_1	15	12	18
S_2	9	14	10
S_3	13	4	26

max 15, max 14, max 26

→

	N_1	N_2	N_2
S_1	15-15	14-12	26-18
S_2	15-9	14-14	26-10
S_3	15-13	14-4	26-26

Regret matrix

	N_1	N_2	N_3	max
S_1	0	2	8	8
S_2	6	0	16	16
S_3	2	10	0	10

→ minimax regret

S_1 ✓

Equal probability criterion

- No objective evidence of a probability distribution for the states of nature, hence a subjective criterion is used.
  - Equal probabilities to each of the states of nature can be assigned
- The subjective assumption of equal probabilities is known as the Laplace criterion, or the criterion of insufficient reason, in management literature.
- With equal probabilities attached to each state of nature, we revert to decision-making under risk.

Coefficient of optimism (Hurwiez) criterion

- It is also known as a weighted average criterion
  - It is a compromise between the **maximax** and **maximin** decision criteria.
  - It takes both into account by assigning them weights in accordance with the degree of optimism or pessimism
- The Hurwicz approach suggests that the decision-maker must select an alternative that maximizes

$$H\,(\text{Criterion of realism}) = \alpha\,(\text{Maximum in column}) + (1 - \alpha)\,(\text{Minimum in column})$$

Where α is the coefficient of optimism. The decision-maker determines the value of α.

Illustrations

	W_1	W_2	W_3	max	min
S_1	15	12	18	18	12
S_2	9	14	10	14	9
S_3	13	4	26	26	4

Optimism $\Rightarrow$ Maximax $\Rightarrow 26 \Rightarrow S_3$

Pessimism $\Rightarrow$ maximin $\Rightarrow 12 \Rightarrow S_1$

Let $\alpha = 0.7$

$S_1 = (18 \times 0.7) + (0.3 \times 12) = 16.2$

$S_2 = (14 \times 0.7) + (0.3 \times 9) = 12.5$

$S_3 = (26 \times 0.7) + (0.3 \times 4) = 19.4$ ✓

Class exercise 5

The following matrix gives the payoff of different strategies (alternatives) A, B, and C against conditions (events) W, X, Y, and Z. Identify the decision taken using the following criteria:

a) Pessimistic
b) Optimistic
c) Equal probability
d) Regret
e) Hurwicz criterion. The decision maker's degree of optimism (α) is 0.7

	W	X	Y	Z
	Ugs	Ugs	Ugs	Ugs
A	4 000	-100	6 000	18 000
B	20 000	5 000	400	0
C	20 000	15 000	-2 000	1 000

Practice Exercise 1

A food products company is contemplating the introduction of a revolutionary new product with new packaging or replacing the existing product at a much higher price (S_1). It may even make a moderate change in the composition of the existing product, with a new packaging at a small increase in price (S_2), or may make a small change in the composition of the existing product, backing it with the word 'New' and a negligible price increase (S_3). The three possible states of nature or events are: (i) a high increase in sales (N_1), (ii) no change in sales (N_2), and (iii) a decrease in sales (N_3). The marketing department of the company worked out the payoffs in terms of yearly net profits for each of the strategies of the three events (expected sales). This is represented in the following table:

	N_1	N_2	N_3
S_1	7 000 000	3 000 000	1 500 000
S_2	5 000 000	4 500 000	0
S_3	3 000 000	3 000 000	3 000 000

Which strategy should the concerned executive choose based on (a) Maximin criterion (b) Maximax criterion (c) Minimax regret criterion and (d) Laplace criterion (assuming $\alpha = 1/3$)?

Practice Exercise 2

An investor is given the following investment alternatives and percentage rates of return

	Low	$Medium$	$High$
$Regular\ shares$	7%	10%	15%
$Risky\ shares$	-10%	12%	25%
$Property$	-12%	18%	30%

Over the past 300 days, 150 days have had medium market conditions, and 60 days have had high market increases. On the basis of these data, state the optimum investment strategy for the investment.

Practice Exercise 3

1. The following matrix gives the payoff (in Rs) of different strategies (alternatives) S_1, S_2 and S_3 against conditions (events) N_1, N_2, N_3 and N_4.

Strategy	State of Nature			
	N_1	N_2	N_3	N_4
S_1	4,000	– 100	6,000	18,000
S_2	20,000	5,000	400	0
S_3	20,000	15,000	– 2,000	1,000

Indicate the decision taken under the following approaches: (i) Pessimistic, (ii) Optimistic, (iii) Equal probability, (iv) Regret, (v) Hurwicz criterion, the degree of optimism being 0.7.

2. In a toy manufacturing company, suppose the product acceptance probabilities are not known but the following data is known:

Product Acceptance	Anticipated First Year Profit ('000 Rs) Product Line		
	Full	Partial	Minimal
Good	8	70	50
Fair	50	45	40
Poor	– 25	– 10	0

Determine the optimal decision under each of the following decision criteria and show how you arrived at it: (a) Maximax, (b) Maximin, (c) Equal likelihood and (d) Minimax regret?

3. The following is a payoff (in rupees) table for three strategies and two states of nature:

Strategy	State of Nature	
	N_1	N_2
S_1	40	60
S_2	10	– 20
S_3	– 40	150

Select a strategy using each of the following decision criteria: (a) Maximax, (b) Minimax regret, (c) Maximin, (d) Minimum risk, assuming equiprobable states.

4. Mr Sethi has Rs 10,000 to invest in one of three options: A, B or C. The return on his investment depends on whether the economy experiences inflation, recession, or no change at all. The possible returns under each economic condition are given below:

Strategy	State of Nature		
	Inflation	Recession	No Change
A	2,000	1,200	1,500
B	3,000	800	1,000
C	2,500	1,000	1,800

What should he decide, using the pessimistic criterion, optimistic criterion, equally likely criterion and regret criterion?

Questions?