capital asset pricing model (CAPM)

Definition

The capital asset pricing model (CAPM) for a security is a linear statistical relationship between the expected excess return of the security and the expected excess return of the market, where expected excess return of a security (market) is defined as the expected return of the security (market) minus the return of a risk-free asset.

Let

- R_i = return of security i
- R_M = return of the market portfolio
- R_f = return of the risk-free asset.

The capital asset pricing model (CAPM) for security i is:

$$E(R_i) - R_f = \beta_i(E(R_M) - R_f);$$

that is,

$$\Gamma_i = \beta_i \Gamma_M,$$

where $\Gamma_M = E(R_M) - R_f$ is the market risk premium, and $\Gamma_i = E(R_i) - R_f$ is the risk premium for security i.

The CAPM can also be written as:

$$E(R_i) = R_f + \beta_i(E(R_M) - R_f);$$

that is,

$$E(R_i) = R_f + \beta_i \Gamma_M.$$

Abstract

The capital asset pricing model (CAPM) for a security is a linear relationship between the expected excess return of the security and the expected excess return of the market. It was developed by William Sharpe, John Lintner and Jan Mossin. It is a useful framework to discuss idiosyncratic and systematic risk. The security market line is a powerful graphical construct of the CAPM. While the CAPM has strong underlying assumptions, recent research has relaxed many of these assumptions. It is commonly used to calculate cost of capital and required rate of return.

History

Bodie, Kane and Marcus (2008: 293) explained that '[t]he capital asset pricing model is a set of predictions concerning equilibrium expected return on risky assets. Harry Markowitz laid down the foundations of modern portfolio management in 1952. The CAPM was developed 12 years later in articles by William Sharpe, John Lintner and Jan Mossin.' See also Brealey and Myers (2003), Damodaran (2002), Markowitz (1999), Miller (1999), and Sharpe (1964).

Idiosyncratic risk

Idiosyncratic risk of security i is defined as:

$$IR_i = SD((R_i - R_f) - \beta_i(R_M - R_f))$$

$$= SD(R_i - \beta_i R_M - (1 - \beta_i)R_f).$$

Consider a stock i that is underpriced (overpriced) according to the information available to an arbitrageur. In order to exploit this profitable opportunity, the arbitrageur will construct the following arbitrage portfolio, if the arbitrageur were constrained to using only the market index and the risk-free asset – see Bhattacharya and O'Brien (2015) for a discussion of the possibilities when a wider set of securities is available to the arbitrageur:

- Arbitrage numerator: go long (short) on the mispriced stock – let's say by $1, which is purely a normalization.
- Other arbitrage legs:
  - Go short (long) α_M on the market index; and
  - Go short (long) α_f on the risk-free asset.

The total amount on these legs has to add up to $1 short (long); and how much to go short (long) on each leg is called the corresponding *hedge ratio*. The ratios α_M and α_f are the relevant hedge ratios, and the *zero-net-investment condition* will require $1-\alpha_M-\alpha_f=0$. The return on period i of this hedge portfolio is $R_{i,t} - \alpha_M R_{M,t} - \alpha_f R_{f,t} = \eta_{i,t}$, which combined with the zero-net-investment condition $1-\alpha_M-\alpha_f=0$ yields $(R_{i,t} - R_{f,t}) = \alpha_M(R_{M,t} - R_{f,t}) + \eta_{i,t}$.

The ordinary least squares (OLS) estimates $\widehat{\alpha_M}$ and $\widehat{\alpha_f} = 1 - \widehat{\alpha_M}$ give us the hedge ratios that optimize the assumed objective of minimum standard deviation, among zero-net-investment portfolios consisting of the market index and the risk-free asset (this simplification is due to Wurgler and Zhuravskaya (2002)). $SD(\widehat{\eta_{i,t}})$ is the idiosyncratic risk of the stock i (i.e., idiosyncratic risk is the *ex post* holding cost for an arbitrageur who is constrained to using only the market index and the risk-free asset as legs of the arbitrage portfolio). See Pontiff (2006) for a detailed discussion on idiosyncratic risk as a holding cost of arbitrage.

Systematic risk

Systematic risk (or non-diversifiable risk or market risk) of security i is defined as:

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$$SR_i = \beta_i = \frac{Cov(R_i, R_M)}{Var(R_M)}.$$

Although this market beta is widely used as a measure of the 'riskiness' of a security, it is actually a measure of how the security's return varies with the market, not of the riskiness of the security *per se*. However, for an investor holding the market portfolio, β_i measures the marginal effect on risk of an increment in the holding of security *i*, everything else remaining the same.

Special cases

Risk-free instrument
When the security *i* is a risk-free instrument (e.g., proxied by a t-bill), β_i=0.

Market portfolio
When the security *i* is the market portfolio (e.g., proxied by a market index), β_i=1.

It is often argued that a beta above one signifies an asset of above-average riskiness, whereas a beta below one signifies an asset of below-average riskiness. Also, a riskier security will have a higher beta and will be discounted at a higher rate. CAPM is consistent with the risk-averse investor's demanding a higher expected return for a riskier asset.

Security market line

The horizontal axis represents beta, and the vertical axis represents expected return. When the CAPM is plotted along these coordinates, the resulting graph is called the security market line. The vertical intercept of the *security market line* is the nominal risk-free rate of return, and its slope is the market risk premium. The value of the security market line corresponding to β=1 is the expected return of the market portfolio.

The security market line is a useful tool to determine whether an asset offers a reasonable expected (or 'fair') return. If the plot of a security is above the security market line, the security is undervalued; similarly, if the plot is below the security market line, the security is overvalued. The expected return of a security minus the expected return on the security market line corresponding to its beta is referred to as the alpha (α) of the securityã therefore, $\alpha_i > 0$ for an undervalued security *i*, and $\alpha_i < 0$ for an overvalued security *i*.

Under CAPM, in equilibrium, expected α_i=0 for each security *i*. However, we find that, on average, low-beta securities have positive alphas and high-beta securities have negative alphas.

It can be argued that security analysis is about identifying securities with non-zero alphas–an investor (or a fund manager) would increase the weights of securities with positive alphas and reduce the weights of securities with negative alphas. Such behavior would increase the price of securities with positive alphas and reduce the price of securities with negative alphas, which would exert pressure in the direction of equilibrium with zero alphas.

It is sometimes important to compare the CAPM against independent estimates of the returns of the security – such independent estimates include comparables analysis. As with any other technique, CAPM would be *ex post* correct if the estimated price equaled the discounted sum of cash flows accruing to the security.

Suppose there are *N* sources of extra-market risk (e.g., industry, inflation) for which there are *N* associated hedge portfolios with returns $R_1, \ldots R_N$. Then, the multi-index form of the CAPM is

$$E(R_i) - R_f = \beta_{iM}(E(R_M) - R_f) + \sum_{n=1}^{N} \beta_{in}(E(R_n) - R_f).$$

Fischer Black derived a more general version of the CAPM in 1972 – in this version, the expected return of an asset in excess of the zero-beta return, is linearly related to its beta. The zero-beta portfolio is the portfolio with the minimum variance of all portfolios uncorrelated with the market portfolio. For the Black version, returns are generally stated on an inflation-adjusted basis.

Assumptions and limitations

The following assumptions are made under CAPM:

- Investors are rational and risk-averse mean-variance optimizers. Investors prefer higher-mean and lower-risk investments. Standard deviation or variance is assumed to be an adequate measure of risk. This is true if normality of returns holds, but may not reflect more general measures of risk and attitudes toward risk.

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- Investors are myopic (i.e., they only plan for one holding period). This is a strong assumption, but it can be relaxed for inter-temporal decision making.
- Investors hold diversified investments. In particular, each investor holds a combination of the market portfolio, where the proportion of each asset equals the market capitalization of the asset divided by the market capitalization of all assets, and the risk-free asset. The market portfolio will be on the efficient frontier. However, the amount invested in the market portfolio by an investor will depend on the investor's wealth and attitude toward risk. A market index is an incomplete proxy for the market portfolio.
- All assets are publicly traded and perfectly divisible. This assumption rules out investments in non-traded assets, such as human capital and private enterprises.
- There are many investors, all of whom are price-takers (i.e., they take prices as given). In other words, each investor's wealth is not significant enough to enable the investor to influence prices by her/his actions. This is the assumption underlying perfect competition in microeconomics.
- Investors can borrow and lend at the risk-free rate – this assumption can be relaxed without affecting the qualitative nature of the arguments.
- Homogeneous EXPECTATIONS hold (i.e., each investor views her/his investment opportunities in an identical manner). In other words, investors are identical except for potentially different wealth and potentially different attitudes toward risk. This is a strong assumption.
- There are no transaction costs and no tax implications. This is also a strong assumption, because trades involve transaction costs such as commissions and fees that depend on frequency and size of trades. Taxes depend on whether the income is from interest, dividends, or capital gains, and investors are in different tax brackets. This assumption can also be somewhat relaxed.

Like any other model, CAPM has its drawbacks, especially as a predictor of actual investment behavior by firms, but its applicability and simplicity make it a useful and popular model of risk and return. An implication of CAPM is that there would be no trades in equilibrium, since investors are assumed to be homogeneous. CAPM is not consistent with size and value effects captured by the Fama and French (1992) three-factor model:

Investors in the CAPM world care only about the systematic or undiversifiable risk of a company and its investments, and not about the portion of the variance of a firm's returns that does not co-vary with the market. Since the market can diversify away all diversifiable risk, the theory implies that firms should have no internal diversification needs … Diversified firms, however, not only exist but also are large and numerous. The CAPM does not help to explain their presence. Nor does the model always provide guidance to the diversified firm that seeks to evaluate an investment project whose risk differs from that of the firm as a whole. Presumably, the firm could estimate the systematic risk of an individual project by using the beta of a single-product firm that undertakes investments similar to the one contemplated by the diversified firm … [t]his is more easily said than done. Consider the extreme case where the firm is deciding whether to invest in research and development for a completely new product… First, there is no comparable single-product firm beta available. Second, the beta for the entire existing firm may be the incorrect one to use for the new product. (Helfat, 1988: 7–8)

[N]ot only might the objective function of the firm differ from that implied by the CAPM but also information problems might cause firms to have difficulty using the CAPM to evaluate potential investments … [I]mperfect information may well influence the way in which managers evaluate project-specific risks … To obtain the capital market risk-adjusted required rate of return for a project, the firm must know the systematic risk of an individual project; this requires the firm to know the covariance between the project's return and the market return. (Helfat, 1988: 12)

The portfolio selection model would be expected to produce different results, both in form and content, than … the CAPM. The form differs primarily in the following two aspects. First, the portfolio selection model yields shares of the investment portfolio allocated to different investments; the CAPM provides information about which projects to undertake but not in what proportions. Second, the comparative spending predictions of the portfolio model … indicate changes in the shares of the portfolio devoted to different investments, rather than absolute spending changes. The CAPM may provide information on the direction of absolute spending changes between periods if projects change from acceptable to unacceptable (or vice versa) but does not indicate changes in investment expenditure shares of the firm's total budget

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> … Most important, the portfolio selection model and the CAPM emphasize different types of risk. The portfolio selection model focuses on covariance risk between firm-level investments, whereas the CAPM focuses on covariance risk between the firm's investments and the market. (Helfat, 1988: 30)

CAPM can provide the required rate of return for a firm's projects – this provides the 'internal rate of return' or the minimum 'hurdle rate' that a project has to yield in order for the project to be acceptable to investors, given the beta of the firm.

CAPM can also be used to set prices for regulated utilities. Given the beta of a regulated utility, CAPM can provide the fair rate of return that investors should get. The rate-setting body can set prices at levels that would generate that level of return for investors.

The CAPM is widely used to estimate a firm's cost of capital. Public estimates of beta – the covariance between the returns on the firm's stock and the returns on a market index such as the S&P 500 – are readily available.

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See also

ARBITRAGE AND ITS LIMITS; CAPITAL STRUCTURE; EXPECTATIONS; HEDGING STRATEGIES; RISK AND UNCERTAINTY

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